FLEXURAL-TORSIONAL-DISTORTIONAL ANALYSIS OF DOUBLE SPINE MONO-SYMMETRIC BOX GIRDER SECTION

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Abstract

The differential equations for torsional-distortional analysis of a double spine mono-symmetric box girder section were obtained following Vlasov's theory. The evaluation of Vlasov's coefficient of the governing differential equations formed a major part of this work. This involved the valuation of the various strain mode diagrams for the box section and an assessment of the strain mode interaction to ascertain the relevant strain modes that affect torsional and distortional displacements. Vlasov's coefficients for the governing equations of equilibrium were obtained using Morh's integral for displacement computations diagram multiplication of the strain modes diagram. By substituting the coefficients into the governing equilibrium equation, a set of coupled fourth order differential equations of equilibrium were obtained. The solution of these equations by method of Fourier sine series gave the flexural torsional and distortional displacements of the 20m simply supported box girder section. The maximum torsional-distortional and flexural displacements were 2.972mm, and 8.773mm respectively.All the strain mode interaction were considered in flexural-torsion-distortional analysis. The maximum torsional-distortional and flexural displacements were 1.448mm, 8.288mm, and 22.186mm respectively.

Keywords:- *Box girder, Distortion, Double spine, Mono-symmetric*

I. PREAMBLE

Thin-walled box girder structures resist eccentric loads in bending action and torsion. The torsional component of the eccentric loads on such structures gives rise to pure torsional stress (Saint Venanttorsion), distortional stress, lateral flexural stress (about the non-symmetric axis of the box girder section) and rotational stress, (Osadebe and Chidolue, 2012). Consequently, the following strain modes are generated in the structure

1. Strain mode due to bending about major (horizontal) axis, in plane and out of plane, $\varphi_1\Psi_1$
2. Strain mode due to bending about minor (vertical) axis, in plane and out of plane, $\varphi_2\Psi_2$
3. Strain mode due to distortion of the cross section, φ_3, Ψ_3
4. Strain mode due to pure rotation or Saint Venant torsion, Ψ_4

It is important to note that $\varphi_1, \varphi_2$and φ_3 represent out of plane displacements, while Ψ_1, Ψ_2, Ψ_3 and Ψ_4 represent inplane displacements. Ideally, torsional-distortional analysis of thin-walled box girder structures should involve investigation of the interaction of the various strain modes given above. However, because of the difficulties and lengthy computations involved, designers and researchers tend to neglect the effect of the strain modes interactions in the analysis of thin walled structures. Most of the researchers whoattempted to consider the interaction effect ended at strain modes in torsion and distortion, φ_2 andφ_3. (Osadebe and Chidolue 2012).

In each analysis method, the structure is simplified by means ofassumptions in the geometry materials and relationship between its components.The accuracy of the structural analysis is dependent upon the method and its assumptions fortunately structural designers are careful enough not to ignore the effect of torsion on a structural member. The effect of warping and distortion on a structural component are however poorly evaluated or ignored in the analysis because of the rigorous analysis involved in their evaluation. There is therefore the need to develop a simple analytical model to enable designers put into consideration the primary, condition of cross sectional deformation in the analysis and design of box girder structures. Thus, a better and more elaborate assessment of all the effect of loads on a thin-walled box girder bridge structure can be achieved by a consideration of the effect of warping torsion and distortion of thin walled double spine mono symmetric box girder bridge

II. REVIEW OF PAST WORK

Several investigators; Chidolue et al (2015), Osadebe and Chidolue (2012), Chidolue and Osadebe (2012), and Mbachu and Osadebe (2013), considered torsional, distortional and flexural stresses in thin-walled mono symmetric box girder structures involving single cell and multicell sections on the other hand, Xian and Xu (2014) Sarode and Vesmawala (2014), Ozgur (2017) and Rubeena (2010), considered horizontally curved reinforced concrete box girder bridges for torsional-distortional and flexural stresses while Qiao (2009), Aric and Granata (2016), considered horizontally curved prestressed concrete box girder structures. Arici et al (2015) considered horizontally curved steel box girder structures. Eze (2010) studied reinforced concrete box column based on the numerous literatures consulted in the literature survey the following observations and comments can be made.

1. Research work done on thin-walled box girder structures covers essentially single cell box girder structure and multi-cell box girder structure either straight or curved.
2. Literature on multiple spine box girders appears to be scarce. Thus, there appears to be a dearth of information on the torsional-distortional behaviour of thin-walled double spine box girder bridge structure.

III. STRAIN MODES

From the energy formulation of the equilibrium it was noted that φ and Ψ represent generalized warping and distortional strain modes respectively.$\varphi_i(s)$ and $\Psi_k(s)$ are elementary displacements) respectively. It was also noted that these displacements are chosen among all displacements possible and are called the generalized strain coordinates of a strip frame. Thus, Vlasov's coefficients of differential equations of equilibrium, which involve a combination of these elementary displacements and their derivatives may be obtained by consideration of the box girder bridge cross section as a strip frame and then applying unit displacement one after the other at the nodal points of the frame in longitudinal direction, to determine the corresponding out of plane displacements at the joints in **n** possible transverse directions, the corresponding transverse (in-plane) displacements can also be obtained. The first order derivatives of these displacement functions may be obtained by numerical differentiation and used for computation of the coefficients with the aid of Morh's integral for displacement computations.

Consideration of the double spine mono-Symmetric strip frame in fig 1.shows that it has eight degrees of freedom in the longitudinal direction and seven in the transverse direction., where in this case m = 8 and n = 7, it follows that we have fifty-six displacement quantities to compute and hence, fifty-six differential equations of distortional equilibrium will be required.The application of Vlasov's generalized strain modes as modified by Varbanov (1970) reduces the number of displacement quantities and hence the differential equations of equilibrium required to solve for them to seven, irrespective of the number of degrees of freedom possessed by the structure.

In the generalized strain modes, there are three strain fields in the longitudinal direction φ_1,φ_2, and φ_3.

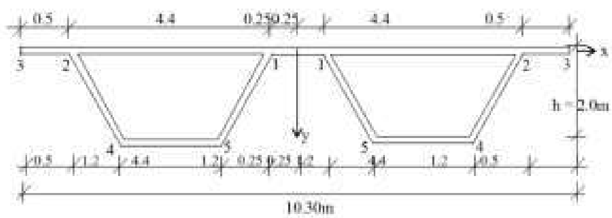


Fig.1 Double Spine Mono-Symmetric Box Girder Section

we have $\varphi(x,s) = \varphi_1(x)\,\varphi_1(s) + \varphi_2(x)\,\varphi_2(s) + \varphi_3(x)\varphi_3(s)$Or

$\varphi(x,s) = \sum_{i=1}^{3} \varphi_i(x)\,\varphi_i(s)$ (20a)

In the transverse direction four strain modes are also recognizedΨ_1,Ψ_2 andΨ_3. Thus, we have $\Psi(x,s) = \Psi_1(x)\,\Psi_1(s) + \Psi_2(x)\,\Psi_2(s) + \Psi_3(x)\Psi_3(s) + \Psi_4(x)\Psi_4(s)$Or

$\Psi(x,s) = \sum_{k=1}^{4} \Psi_k(x)\,\Psi_k(s)$ (20b)

Where φ_1 = out of plane displacement parameter when the load is acting (vertically) normal to the top flange of the girder, i.e. bending is about horizontal axis.

φ_2 = out of plane displacement parameter when the load is acting tangential to the plane of the flanges i.e. bending is about vertical axis.

φ_3= out of plane displacement parameter due to distortion of the cross section i.e; the warping function.

Ψ_1 = In-plane displacement parameter due to the load giving rise to φ_1

Ψ_2 = In-plane displacement parameters due to the load giving rise to φ_2

Ψ_3= In-plane displacement parameter due to the distortion of the cross section i.e non uniform torsion.

Ψ_4 In-plane displacement functions due to pure rotation or Saint Venant torsion of the cross section.

IV. STRAIN MODE DIAGRAMS

Consider a simply supported girder loaded as shown in Fig.2a. if we assume the normal bean theory, i.e.; neutral axis remaining neutral before and after bending then the distortion of the cross section will be as shown in Fig.2 where,$\theta$is the distortion angle (rotation of the vertical axis). The displacement φ_1 at any distance R, from the centroid is given by φ_1 = Rθ. If we assure a unit rotation of the vertical (z) axis then φ_1 = R , at any point on the cross section. Note that φ_1 can be positive or negative depending on the value of R, in the tension or compression zone of the girder. Thus, $\varphi_1$is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.

Similarly, if the load is acting in a horizontal (y-y) direction, normal to the x-z plane in Fig.2, then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2 , displacement out of plane. The values of φ_2 , are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian.

The warping function $\varphi_{3,}$ of the beam cross section is obtained as detailed in Fig.3a it has been explained that the warping function is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either x or y direction and without longitudinal extension. Ψ_1 and Ψ_2 are in-plane displacement of the cross section in x-z and x-y planes respectively while Ψ_3 is the distortion of the cross section. They can be obtained by numerical differentiation of $\varphi_{1,}$ $\varphi_{2,}$and φ_3 diagrams respectively.

Ψ_4 is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, Ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. It is assumed to be positive if the member moves in the positive directions of the coordinate axis and negative otherwise.

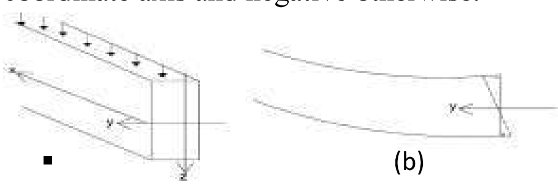
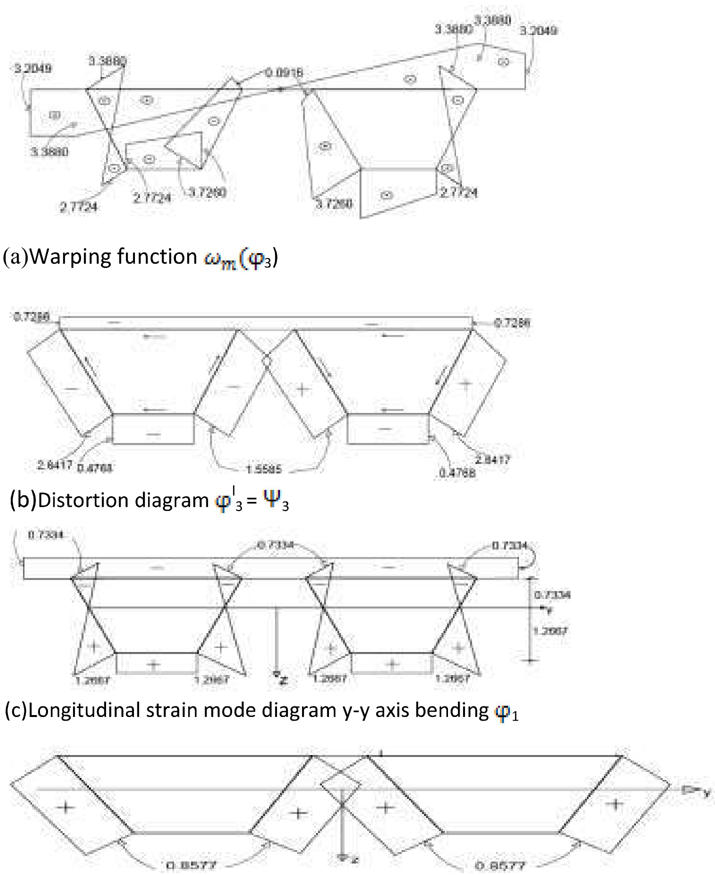


Fig.2 Simply Supported Girder Section and Cross Section Distortion

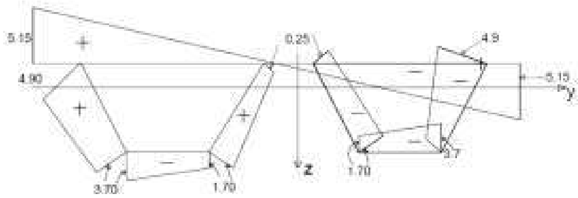


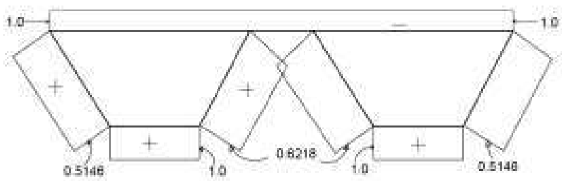
(a)Warping function $\omega_m(\varphi_3)$

(b)Distortion diagram $\varphi'_3 = \Psi_3$

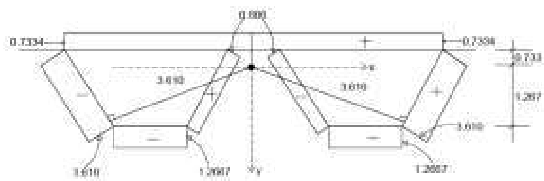
(c)Longitudinal strain mode diagram y-y axis bending φ_1

(d)Transverse strain mode in y directionΨ_1





(f)Transverse strain mode in z direction Ψ_2



(g) Rotation strain mode , Ψ_4 Fig.3 Generalized Strain Mode Diagram

For mono-symmetric section, the relevant Vlasov's coefficients for Torsional-distortional equilibrium are a_{33}, $b_{33} = r_{33}$, $r_{34} = r_{43}$, r_{44}

$a_{33} = \int \varphi_3(s)\, \varphi_3(s)\, dA = 24.682$

$b_{33} = \int \varphi^1_3 \quad \varphi^1_3 (s)\, dA = 9.918$

$r_{34} = \int \Psi_{3s} \ \Psi_{4s}\ dA = 7.107$

$r_{44} = \int \Psi_4 \Psi_4 dA = 15.33$

Note,

$b_{33} = C_{33} = r_{33} = 9.918$

$r_{34} = r_{43} = 7.107$

The coefficient $S_{hk} = S_{kh} = \frac{I}{E}\int \frac{M_3(S) M_3(S)}{EIs}$ (21)

Where M_3 (s) is the distortional bending moment

V. DETERMINATION OF DISTORTIONAL BENDING MOMENT FOR THE BOX GIRDER

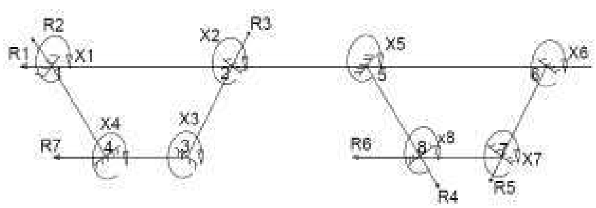


Fig. 4 Base System for Evaluation of Distortion Bending Moment

Fig.3a shows the base system for the evaluation of distortional bending moment for the double spine mono-symmetric box girder. The evaluation of the distortional bending moment involves the application of unit rotation $X_1$to X_8at joint 1 to 8 respectively and applying unit transverse displacement of joints based on distortion diagram

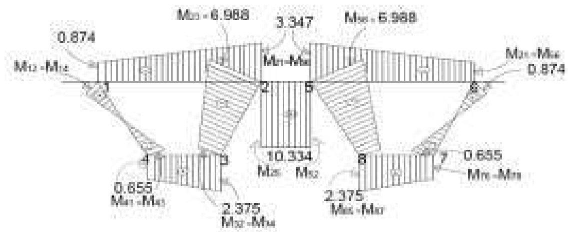


Fig.5 Bending Moment Diagram Due to Distortion of the Cross Section

Shk = shk = $\frac{1}{E}\int\frac{M_k(s)M_h(s)}{EIs}$ dsShk = Skh = S_{33} = $2.891\text{x}10^{-3}I_s$only S_{33}has value

VI. FORMULATION OF DIFFERENTIAL EQUATIONS OF EQUILIBRIUM

The relevant coefficients for torsional-distortional equilibrium are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{43}, r_{44}, and s_{33}. Substituting these into the matrix notation equation (8) and (9) we obtain:

$$k\begin{bmatrix}0&0&0\\0&0&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}U_1''\\U_2''\\U_3''\end{bmatrix}-\begin{bmatrix}0&0&0\\0&0&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}U_1\\U_2\\U_3\end{bmatrix}-\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&c_{33}&c_{34}\end{bmatrix}\begin{bmatrix}V_1'\\V_2'\\V_3'\\V_4'\end{bmatrix}=0$$

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&C_{33}\\0&0&C_{43}\end{bmatrix}\begin{bmatrix}U_1'\\U_2'\\U_3'\end{bmatrix}-K\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&S_{33}&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}+$$

$$\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&r_{33}&r_{34}\\0&0&r_{43}&r_{44}\end{bmatrix}=-\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}$$

Multiplying out we obtain

$Ka_{33}U_3'' - b_{33}U_3 - C_{33}V_3' - C_{34}V_4' = 0$

$C_{33}U_3' - KS_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G}$

$C_{43}U_3' - r4_3V_3'' + r_{44}V_4'' = -\frac{q_4}{G}$

Simplifying further we obtain

$\beta_1 V_4'' - \gamma_1 V_3 = K_1$ (22a)

$\propto_1 V_3^{iv} + \propto_2 V_4^{iv} - \beta_2 V_4'' = K_2$ (22b)

Where $\propto_1 = ka_{33}c_{43}$; $\propto_2 = ka_{33}r_{44}$;

$\beta_1 = r_{34}c_{43} - c_{33}r_{44}$

$\beta_2 = b_{33}r_{44} - c_{34}c_{43}$;

$\gamma_1 = c_{43}ks_{33}$ (23)

$K_1 = -C_{33}\frac{q_4}{G} - C_{43}\frac{q_3}{G}$; $K_2 = \left(\frac{b_{33}q_4}{G}\right)$ (24)

Torsional – Distortional Analysis of Mono-Symmetric Box Girder Structure

In this section the solutions of the differential equations of equilibrium are obtained for the double spine mono-symmetric box girder section shown in fig.1. Live loads are considered according to AASHTO-LRFD following the HL-93 loading. Uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A two span simply supported bridge deck structure, 20m per span, was considered.

The obtained torsional loads are as follows

$q_3 = 1410.318KN$, $q_4 = 3732.202$ KN

Parameters for the governing equations (22a and 22b) are:

$\alpha_1 = K\ a_{33}\ C_{43}$; $\alpha_2 = Ka_{33}\ r_{44}$

$\beta_1 = r_{34} C_{43} - C_{33}\ r_{44}$; $\beta_2 = b_{33}\ r_{44} - C_{34}\ C_{43}$

$\gamma_1 = C_{43}\ K\ S_{33}$; $K_1 = C_{33}\frac{E4}{G} = C_{43}\frac{E3}{G}$

$K_2 = \frac{b_{33}q_4}{G}$; $S_{33} = 2.891 \times 10^{-2} I_s$

$k = 2\ (1 + V)$; $k = 2\ (1 + 0.25) = 2.5$; $\nu = 0.25$ for concrete

$E = 24 \times 10^9\ N/m^2$; $G = 9.6 \times 10^9\ N/m^2$

$\therefore \alpha_1 = 2.5 \times 24.682 \times 7.107 = 438.537$

$\alpha_2 = 2.5 \times 24.682 \times 15.153 = 935.016$

$\beta_1 = 7.107 \times 7.107 - 9.918 \times 15.153 = -99.778$

$\beta_2 = 9.918 \times 15.153 - 7.107 \times 7.107 = 99.778$

$\gamma_1 = 7.107 \times 2.5 \times 2.891 \times 10^{-2} = 0.5137$

$K_1 = 9.918 \times \frac{3732.202 \times 10^3}{9.6 \times 10^9} - 7.107 \times \frac{1410.318 \times 10^3}{9.6 \times 10^9} = 0.0028109$

$K_2 = \frac{(9.918 \times 3732.202 \times 10^3)}{9.6 \times 10^9} = 3.856 \times 10^{-3}$

Substituting the coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, K_1$ and K_2 We obtain equations (25) and (26) below

$$438.537\ V_3^{IV} + 935.016\ V_4^{IV} - 99.778\ V_4^{II} = 3.856 \times 10^{-3} \quad (25a)$$

$$-99.778 V_4^{II} - 0.5137\ V_3 = 2.811 \times 10^{-3} \quad (25b)$$

Integrating by method of Trigonometric Series with accelerated convergence we obtain

$$V_3\ (x) = 8.773 \times 10^{-3} \sin \frac{\pi x}{20} \quad (26)$$

$$V_4\ (x) = 2.972 \times 10^{-3} \sin \frac{\pi x}{20}$$

Table 1:Variation of torsional and distortional displacements along the length of the girder (20m simply supported)

Distance x from left support (m)	Sin $\frac{\pi x}{20}$	Distortional Displacement $V_3\ (x)$ $\times 10^{-3}$ m	Torsional Displacement $V_4\ (x)$ $\times 10^{-3}$ m
0	0.000	0.000	0.000
2	0.309	2.711	0.918
4.	0.588	5.158	1.747
6.	0.809	7.097	2.404

8.	0.951	8.343	2.826
10	1.000	8.773	2.972
12	0.951	8.343	2.826
14.	0.809	7.097	2.404
16	0.588	5.158	1.747
18	0.309	2.711	0.918
20	0.000	0.000	0.000

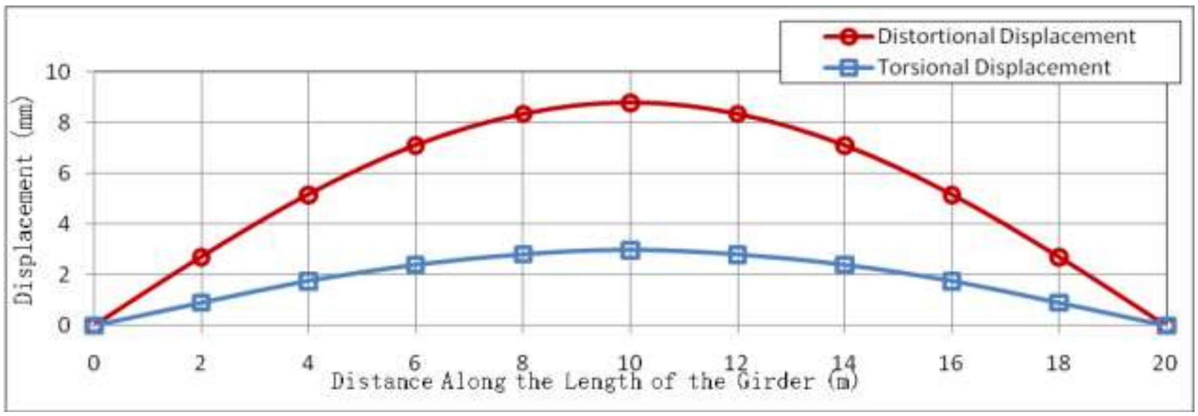


Fig. 6 Variation of torsional and distortional displacement along the length of the girder

VII. FLEXURAL-TORSIONAL-DISTORTIONAL ANALYSIS OF DOUBLE SPINE MONO- SYMMETRIC BOX GIRDER SECTION

The governing equations of equilibrium are as follows.

$$\beta_4 V_2^{ii} + \beta_5 V_3^{ii} + \beta_6 V_4^{ii} - \beta_4 - V_2^{ii} \gamma_4 V_3 = -K \quad (4.31a)$$

$$\alpha_4 V_2^{iv} + \alpha_5 V_3^{iv} + \alpha_6 V_4^{iv} - \beta_7 V_2^{ii} - \beta_8 V_3^{ii} - \beta_9 V_4^{ii} + \gamma_2 V_3 = K_4 \quad (4.31b)$$

$$\alpha_7 V_2^{iv} + \alpha_8 V_3^{iv} + \alpha_9 V_4^{iv} - \beta_{10} V_2^{ii} - \beta_{11} V_3^{ii} - \beta_{12} V_4^{ii} + \gamma_3 V_3 = K_5 \quad (4.31c)$$

The relevant strain modes for flexural distortional and torsional displacement are strain mode 2,3 and 4 respectively $\varphi_2, \varphi_3, \Psi_2, \Psi_3, \Psi_4$ and the relevant Vlasov's coefficients are
$a_{22}, a_{23}, a_{33}, b_{22} =, b_{23} = c_{23} = r_{23}, b_{33} = c_{33} =$
$r_{33}, r_{44}, c_{24} = c_{42} = r_{24}, c_{34} = c_{43} = r_{34} = r_{43}, and\ s_{33}$

$a_{22} = \int \varphi_2 \varphi_2 dA = \sum \varphi_2 \varphi_2 dA.$

$b_{22} = \int \Psi_2 \Psi_2 dA = \sum \Psi_2 \Psi_2 dA.$

$b_{23} = c_{23} = r_{23} = \int \Psi_2 \Psi_3 dA = \sum \Psi_2 \Psi_3 dA.$

$b_{33} = c_{33} = r_{33} = \int \Psi_3 \Psi_3 dA = \sum \Psi_3 \Psi_3 dA.$

$c_{24} = c_{42} = r_{42} = \int \Psi_4 \Psi_2 dA = \sum \Psi_4 \Psi_2 dA.$

$c_{34} = c_{43} = r_{34} = r_{43} \int \Psi_3 \Psi_4 dA = \sum \Psi_3 \Psi_4 dA.$

$a_{23} = \int \varphi_2 \varphi_3 dA = \sum \varphi_2 \varphi_3 dA.$

$a_{22} = \int \varphi_2 \varphi_2 dA = \sum \varphi_2 \varphi_3 dA.$

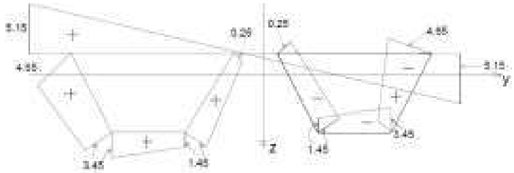


Fig 4.28 Longitudinal strain mode z-z axis bending ϕ_2

The relevant Vlasov's coefficients for flexural –torsional-distortional analysis of the double spine mono-symmetric section are as follows.

$a_{22} = 39.479$, $a_{23} = 6.6404$

$b_{22} = c_{22} = r_{22} = 3.3540$

$b_{23} = c_{23} = r_{23} = 0.1114$

$b_{33} = c_{33} = r_{33} = 9.918$

$c_{24} = c_{42} = r_{24} = 1.8799$

$c_{34} = c_{43} = r_{34} = \gamma_{43} = 7.107$

$r_{44} = 15.33$, $s_{33} = 0.02891$

The coefficients of the equations are:

$$\alpha_1 = \left[\frac{\frac{r_{22}}{c_{22}} - \frac{r_{22}}{c_{22}}}{\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}}\right] = -1 \qquad \alpha_2 = \left[\frac{\frac{r_{24}}{c_{22}} - \frac{r_{24}}{c_{22}}}{\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}}\right] = -0.5369 \qquad \alpha_3 = \left[\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}\right] = 2.421 \times 10^{-4}$$

$$\beta_1 = \left[\frac{\frac{r_{22}}{c_{22}} - \frac{r_{22}}{c_{22}}}{\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}}\right] = -1 \qquad \beta_2 = \left[\frac{\frac{r_{24}}{c_{22}} - \frac{r_{24}}{c_{22}}}{\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}}\right] = -0.7105 \qquad \beta_3 = \left[\frac{c_{22}}{c_{22}} - \frac{c_{22}}{c_{22}}\right] = -0.0072899$$

$\beta_4 = c_{42}\alpha_1 + r_{42} = 0, \ \beta_5 = c_{43}\beta_1 + r_{43}$

$\beta_6 = c_{42}\alpha_2 + c_{43}\beta_2 + r_{44} = 9.2712, \ \beta_7 = b_{22}\alpha_1 + c_{22} = 0$

$\beta_8 = ka_{23}\beta_3 + b_{23}\beta_1 + ka_{22}\alpha_3 + c_{23} = p_8 = -0.11899$

$\beta_9 = (b_{22}\alpha_2 + b_{33}\beta_2 + c_{24}) = -6.9676$

$\beta_{10} = (b_{32}\alpha_1 + c_{32}) = 0$

$\beta_{11} ka_{32} + ka_{32}\beta_3 + b_{33}\beta_1 + c_{33} = -0.17132$

$\beta_{12} = (b_{32}\alpha_2 + b_{33}\beta_2 + c_{34}) = 4.503 \ x \ 10^{-4}$.

$\alpha_4 = ka_{22}\alpha_1 = -98.6975$

$\alpha_5 = ka_{23}\beta_1 = -16.601$

$\alpha_6 = k(a_{22}\alpha_2 + a_{23}\beta_2) = -64.7857$

$\alpha_7 = ka_{32}\alpha_1 = -16.601$

$\alpha_8 = ka_{33}\beta_1 = -61.705$

$\alpha_9 = k(a_{32}\alpha_2 + a_{33}\beta_2) = -52.7545$

$\gamma_1 = (c_{42}\alpha_3 - c_{43}\beta_2) = -0.05135$

$\gamma_2 = (b_{22}\alpha_3 + b_{23}\beta_3) = 9.1474 \ x \ 10^{-7}$.

$\gamma_3 = (b_{32}\alpha_3 + b_{33}\beta_3) = -0.07227$.

$$k_1 = \overset{q_3}{\left[\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right]} c_{32} G \overset{q_2}{\left[\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right]} c_{22} G$$

$k_1 = -7.9531 \ x \ 10^{-6}$.

$$k_1 = \overset{q_2}{\left[\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right]} c_{33} G \overset{q_2}{\left[\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right]} c_{23} G$$

$k_2 = 3.275 \ x \ 10^{-7}$.

$k_3 = \frac{q_4}{G} + c_{42}k_2 + c_{43}k_1 = 6.0871x10^{-3}$.

$k_4 = (b_{22}k_2 + b_{23}k_1) = -9.154 \ x \ 10^{-11}$.

$k_5 = (b_{32}k_2 + b_{33}k_1) = -9.7766 \ x \ 10^{-5}$.

Substituting the coefficients and constants into equation (4.31a, 4.31b and 4.31c)

$$9.2712V_4^{ii} + 5.135x10^{-2}V_3 = -6.0871x10^{-3} \quad (4.32a)$$

$$-98.6975 \ V_2^{iv} - 16.601V_3^{iv} - 64.7856 \ V_4^{iv} + 0.11899V_3^{ii} + 6.9676 \ V_4^{ii} + 9.1474x10^{-7}V_3 = -9.154x10^{-11} \quad (4.32b)$$

$$-16.601 \ V_2^{iv} - 61.705 \ V_3^{iv} - 52.7545 \ V_4^{iv} + 0.17132 \ V_3^{ii} - 4.503x10^{-4} \ V_4^{ii} - 7.227x10^{-2}V_3 = -9.7766x10^{-5} \quad (4.32c)$$

Integrating by method of trigonometric series

Interval = Span L=20, $0 \leq x \leq 20$

Boundary condition

$V_2(0) = 0$; $V_2^{ii}(0) = 0$; $V_2(L) = 0$; $V_2^{ii}(L) = 0$; $V_2(L) = 0$; $V_3(0) = 0$; $V_3^{ii}(0) = 0$; $V_3(L) = 0$

$V_3^{ii}(L) = 0$; $V_4(0) = 0$; $V_4^{ii}(0) = 0$; $V_4(L) = 0$; $V_4^{ii}(L) = 0$

We seek solution in the form

$V_2(x) = f_2(x) + g_2(x)$

$V_3(x) = f_3(x) + g_3(x)$

$V_4(x) = f_4(x) + g_4(x)$

Substituting in equation (4.32a, 4.32b and 4.32c) respectively

$9.2712 f_4^{ii} + 5.135x10^{-2} f_3 + 9.2712\, g_4^{ii} + 5.135x10^{-2} g_3 = -6.0871x10^{-3}$

$-98.6975 f_2^{iv} - 16.601\, f_3^{iv} - 64.7857\, f_4^{iv} + 0.11899\, f_3^{ii} + 6.6976\, f_4^{ii} + 9.147410^{-7} f_3 + 98.6975\, g_2^{iv} - 16.601\, g_3^{iv} - 64.7857\, g_4^{iv} + 0.11899\, g_3^{ii} + 6.6976\, g_4^{ii} + 9.147410^{-7} g_3 = -9.154x10^{-11}$

$-16.601 f_2^{iv} - 61.705\, F_3^{iv} - 52.7545\, F_4^{iv} + 0.17132\, F_3^{ii} - 4.503x10^{-4} F_4^{ii} - 7.227x10^{-2} F_3 - 16.601 g_2^{iv} - 61.705\, g_3^{iv} - 52.7545\, g_4^{iv} + 0.17132 g_3^{ii} - 4.503x10^{-4} g_4^{ii} - 7.227x10^{-2} g_3 =$

$-9.7766x10^{-5}$

$P_1(x) = 9.2712 F_4^{ii} + 5.135x10^{-2} F_3.$

$P_2(x) = -98.6975\, F_2^{iv} - 16.601 F_3^{iv} - 64.7857 F_4^{iv} + 0.11899 F_3^{ii} + 6.6976 F_4^{ii} + 9.1474x10^{-7} F_3$

Also

$P_1(x) = 9.2712 g_4^{ii} + 5.135x10^{-2} g_3.$

$P_2(x) = -98.6975\, g_2^{iv} - 16.601 g_3^{iv} - 64.7857 g_4^{iv} + 0.11899 g_3^{ii} + 6.6976 g_4^{ii} + 9.1474x10^{-7} g_4$

$P_3(x) = -16.601\, g_2^{iv} - 61.705\, g_3^{iv} - 52.7545\, g_4^{iv} + 0.17132 g_3^{ii} - 4.053x10^{-4} g_4^{ii} - 7.227x10^{-2} g_3$

The complementary functions are

$g_2(x);\ g_3(x) and\ g_4(x).$

$g_2(x) = \sum_{n=1}^{\infty} a_{2n} \sin \frac{\alpha n x}{L}.$

$g_3(x) = \sum_{n=1}^{\infty} a_{3n} \sin \frac{\alpha n x}{L}.$

$g_4(x) = \sum_{n=1}^{\infty} a_{4n} \sin \frac{\alpha n x}{L}.$

Where $\alpha n = n\pi$

The auxiliary function is as follows

$F_2(x) = A_o + A_1 x + A_2 x^2 + A_3 x^3.$

$F_3(x) = B_o + B_1 x + B_2 (x)^2 + B_3 x^3.$

$F_4(x) = C_o + C_1 x + C_2 x^2 + C_3 x^3.$

For the simply supported span, the boundary conditions are such that zero $A_0 = A_1 = A_2 = A_3 = B_0 = B_1 = B_2 = B_3 = C_0 = C_1 = C_2 = C_3 = 0$

Therefore,

$F_2(x) = P_2(x) = 0.$

$F_3(x) = P_3(x) = 0.$

$F_4(x) = P_4(x) = 0.$

$V_2(x) = g_2(x).$

$V_3(x) = g_3(x).$

$V_4(x) = g_4(x).$

$9.2712\, g_2^{ii} + 5.13510^{-2} g_3 + 6.087110^{-3} = 0$

$-98.6975\, g_2^{iv} - 16.601 g_3^{iv} - 64.7857$
$g_4^{iv} + 0.11899 g_3^{ii} + 6.9676 g_4^{ii} + 9.1474x10^{-7} g_3 +$
$9.154x10^{-11} = 0$

$-16.601\, g_2^{iv} - 61.705 g_3^{iv} - 52.545 g_4^{iv} + 0.17132 g_3^{ii} +$
$4.503\, x + 10^{-4} g_4^{ii} - 7.227x10^{-2} g_3 + 9.7766x10^{-5}$
$= 0$

$g_2(x) = \sum_{n=1}^{\infty} a_{2n} \sin \quad \frac{\alpha nx}{L}.$

$g_2^{ii}(x) = -\sum_{n=1}^{\infty} a_{2n}\, \alpha_n^2 \sin \quad \frac{\alpha nx}{L}.$

$g_2^{iv}(x) = \sum_{n=1}^{\infty} a_{2n}\, \alpha_n^4 \sin \quad \frac{\alpha nx}{L}.$

$g_3(x) = \sum_{n=1}^{\infty} a_{3n} \sin \quad \frac{\alpha nx}{L}.$

$g_3^{ii}(x) = -\sum_{n=1}^{\infty} a_{3n} \alpha_n^2 \sin \quad \frac{\alpha nx}{L}.$

$g_3^{iv}(x) = \sum_{n=1}^{\infty} a_{3n} \alpha_n^4 \sin \quad \frac{\alpha nx}{L}.$

$g_4(x) = \sum_{n=1}^{\infty} a_{4n} \alpha_n^4 \sin \quad \frac{\alpha nx}{L}.$

$g_4^{ii}(x) = -\sum_{n=1}^{\infty} a_{4n} \alpha_n^2 \sin \quad \frac{\alpha nx}{L}.$

$g_4^{iv}(x) = \sum_{n=1}^{\infty} a_{4n} \alpha_n^4 \sin \quad \frac{\alpha nx}{L}.$

$(-9.2712 \sum a_{4n}\, \alpha_n^2 + 5.135x10^{-2} \sum a_{3n}) \sin\frac{\alpha nx}{L} = -6.0871x10^{-3}.$

$(-98.697 \sum a_{2n}\, \alpha_n^4 - 16.601$
$\sum a_{3n}\, \alpha_n^4 -$
$64.7857 \sum a_{4n} \alpha_n^2 + 9.1474x10^{-7} \sum a_{3n}) \frac{sin\alpha nx}{L} =$
$-9.154x10^{-11}$
.

$(-16.601 \sum a_{2n}\, \alpha_n^4 - 61.705$
$\sum a_{3n}\, \alpha_n^4 - 52.7545 \sum a_{4n} \alpha_n^4 - 0.17132 \sum a_{3n}\, \alpha_n^2 +$
$4.503x10^{-4} \sum a_{4n}\, \alpha_n^2 - 7.227x10^{-2} \sum a_{3n}) \frac{sin\alpha nx}{L} =$
$-9.7766x10^{-5}$

.

$-9.2712\, a_{4n} \alpha_n^2 + 5.13510^{-2} a_{3n} = 6.0871x10^{-3}$

$(-9.2712\ \alpha_n^2)\, a_{4n} + (5.135x10^{-2}) a_{3n} = 6.0871x10^{-3}$

$(-98.6975\ \alpha_n^4)\, a_{2n} - (16.601 \alpha_n^4 + 0.11899) \alpha_n^2 - 9.1474x10^{-3}$

$a_{3n} - (64.7857 \alpha_n^4 + 6.6976 \alpha_n^2) a_{4n} = -9.154x10^{-11}$

$(-16.601\ \alpha_n^4)\, a_{2n} - (61.601 \alpha_n^4 + 0.17132 \alpha_n^2 + 7.227x10^{-2})$

$a_{3n} - (52.7545\ \alpha_n^4 - 4.503x10^{-4} \alpha_n^2) a_{4n} = -9.7766x10^{-5}$

for $n = 1, \alpha_1 = \frac{\pi}{20}, \alpha_1^2 = 2.4674x10^{-2} \alpha_4 = 6.088x10^{-4}$.

Substituting

$(-0.228758) \alpha_{41} + (5.135x10^{-2}) a_{31} = -6.0871x10^{-3}$

$(-6.0087x10^{-2}) \alpha_{21} - (1.3041x10^{-2}) a_{31} - (0.20469) a_{41} = -9.154x10^{-11}$

$(1.01067x10^{-2}) \alpha_{21} - (0.114063) a_{31} - (3.2106x10^{-2}) a_{41} = -9.7766x10^{-5}$

$(5.135x10^{-2}) \alpha_{31} - (0.22876) a_{41} = 6.0871x10^{-3}$

$-(6.0087x10^{-2}) \alpha_{21} - (1.3042x10^{-2}) a_{31} - (0.20469) a_{41} = -9.154x10^{-11}$

$(1.01067x10^{-2}) \alpha_{21} - (0.114063) a_{31} - (3.2106x10^{-2}) a_{41} = -9.7766x10^{-5}$

Solving equation, we obtain

$\alpha_{21} = 0.022186, \alpha_{31} = 0.008288, \alpha_{41} = 0.001448$

$V_2(x) = 22.186x10^{-3} \sin\frac{\pi x}{20}, V_3(x) = 8.288x10^{-3} \sin\frac{\pi x}{20}, V_4(x) = 1.448x10^{-3} \sin\frac{\pi x}{20}$

Table 4.5 Variation of flexural, torsional and distortional displacement along the length of the double spine mono-symmetric box girder section 20m span bridge

Distance X from Left support	Sin $\frac{\pi x}{20}$	$V_2(x)$ x10^{-3}	$V_3(x)$ x10^{-3}	$V_4(x)$ x10^{-3}
0	0.000	0.000	0.000	0.000
2	0.3090	6.855	2.561	0.447
4	0.5878	13.041	4.872	0.8511
6	0.8090	17.948	6.705	1.171

8	0.9511	21.101	7.882	1.377
10	1	22.186	8.288	1.448
12	0.9511	21.101	7.882	1.377
14	0.8090	17.948	6.705	1.377
16	0.5878	13.041	4.872	0.8511
18	0.3090	6.855	2.561	0.447
20	0.000	0.000	0.000	0.000

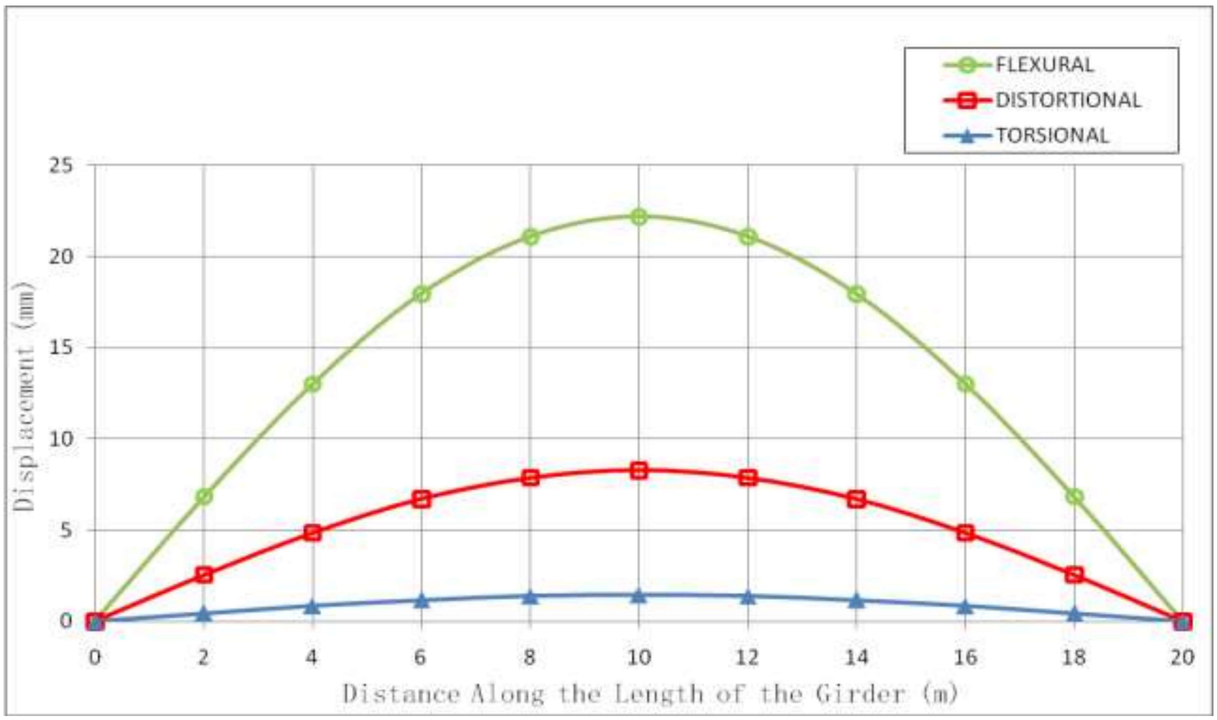


Fig 4.29 Graph of variation of flexural, torsional, distortional displacements along the length of the girder

DISCUSSION OF RESULTS:

The governing differential equations of torsional-distortional equilibrium for the double spine mono-symmetric box girder structures are given by eqn. (25a and 25b).

The solution of the torsional-distortional equations of equilibrium for the double spine mono-symmetric box girder studied is given by:

$$V_3 = 8.773x10^{-3} \sin \frac{\pi x}{L}$$

(26)

$$V_4 = 2.972x10^{-3} \sin \frac{\pi x}{L}$$

Where L represents the span of the girder

The torsional and distortional deformations obtained by integration of eqn. (25) are given by eqn. (26). The results of the analysis are presented in table 1 with graphical presentation in fig.3. The maximum (mid-span) torsional displacement was 2.97mm while the mid-span distortional displacement was 8.77mm. Thus the maximum distortional deformation is about 3 times that of torsional deformation. This explains why torsional stresses may be neglected but not distortional stresses. Flexural-torsional-distortional deformation of the 20m span bridge from left of support increased gradually from zero at the support until it reached a maximum of 22.186 for flexural deformation, 8.288 for distortional deformation and 1.448 for torsional deformation. The maximum displacement occurred at the mid span of the box girder longitudinal span. The flexural-torsional-distortional displacements then gradually decrease to zero at the right hand support. Table 4.5 shows the variation of flexural-torsional-distortional displacement along the length of the double spine monosymmetric box girder section of 20m span while fig. 4.29 shows the graphical representation of the displacements plotted along the span from left to right.

CONCLUSION:

The distortional deformations were found to be about three times that of torsional deformation.
The response of double spine mono-symmetric box girder structure to torsional and distortional loads is similar to that of single and multi-cellular box girders obtained from earlier studies by other researchers; Chidolue and Osadebe 2012.

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